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Once again about beam-size or MD-effect at colliding beams

G.L. Kotkin and V.G. Serbo*

Novosibirsk State University, 630090, Novosibirsk, Russia

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Abstract

For several processes at colliding beams, macroscopically large impact parameters give an essential contribution to the standard cross section. These impact parameters may be much larger than the transverse sizes of the colliding bunches. In that case, the standard calculations have to be essentially modify. The corresponding formulae were given twenty years ago. In recent paper of Baier and Katkov [17] it was claimed that the previous results about bremsstrahlung spectrum have to be revised and an additional subtraction related to the coherent contribution has to be done. This additional term has been calculated with the result that it may be essential for the performed and future experiments. In the present paper we analyze in detail the coherent and incoherent contributions in the conditions, considered in paper [17]. In contract to above claims, we found out that under these conditions the coherent contribution is completely negligible and, therefore, there is no need to revise the previous results.

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I. INTRODUCTION

A. Beam-size or MD-effect

The so called beam-size or MD-effect is a phenomenon discovered in experiments [1] at the MD-1 detector (the VEPP-4 accelerator with e^+e^- colliding beams , Novosibirsk 1981). It was found out that for ordinary bremsstrahlung, macroscopically large impact parameters should be taken into consideration. These impact parameters may be much larger than the transverse sizes of the interacting particle bunches. In that case, the standard calculations, which do not take into account this fact, will give incorrect results. The detailed description of the MD-effect can be found in review [2].

In 1980–1981 a dedicated study of the process $e^+e^- \to e^+e^-\gamma$ has been performed at the collider VEPP-4 in Novosibirsk using the detector MD-1 for an energy of the electron and positron beams $E_e = E_p = 1.8$ GeV and in a wide interval of the photon energy E_γ from 0.5 MeV to $E_\gamma \approx E_e$. It was observed [1] that the number of measured photons was smaller than that expected. The deviation from the standard calculation reached 30% in the region of small photon energies and vanished for large energies of the photons. A. Tikhonov [3] pointed out that those impact parameters ϱ, which give an essential contribution to the standard cross section, reach values of $\varrho_m \sim 5$ cm whereas the transverse size of the bunch is $\sigma_\perp \sim 10^{-3}$ cm. The limitation of the impact parameters to values $\varrho \lesssim \sigma_\perp$ is just the reason for the decreasing number of observed photons.

The first calculations of this effect have been performed in Refs. [4] and [5] using different versions of quasi–classical calculations in the region of large impact parameters. Further experiments, including the measurement of the radiation probability as function of the beam parameters, supported the concept that the effect arises from the limitation of the impact parameters. Later on, the effect of limited impact parameters was taken into account when the single bremsstrahlung was used for measuring the luminosity at the VEPP–4 collider [6] and at the LEP-I collider [7]. In the case of the VEPP–4 experiment [6], it was checked that the luminosities obtained using either this process or using other reactions (such as the double bremsstrahlung process $e^+e^- \to e^+e^-\gamma\gamma$, where the MD-effect is absent) agreed with each other.

A general scheme to calculate the finite beam size effect had been developed in paper [8] starting from the quantum description of collisions as an interaction of wave packets forming bunches. Since the effect under discussion is dominated by small momentum transfer, the general formulae can be considerably simplified. The corresponding approximate formulae were given. They are obtained from an analysis of Feynman diagrams and it allows to estimate the accuracy of approximation. In a second step, the transverse motion of the particles in the beams can be neglected. The less exact (but simpler) formulae, which are then found, correspond to the results of Refs. [4] and [5]. It has also been shown that similar effects have to be expected for several other reactions such as bremsstrahlung for colliding ep–beams [9], [10], e^+e^-– pair production in $e^{\pm}e$ and γe collisions [8]. The corresponding corrections to the standard formulae are now included in programs for simulation of events at linear colliders. The influence of MD-effect on polarization had been considered in Ref. [11].

The possibility to create high-energy colliding $\mu^+\mu^-$ beams is now wildly discussed. For several processes at such colliders a new type of beam-size effect will take place — the so called linear beam-size effect [12]. The calculation of this effect had been performed by method developed for MD-effect in [8].

In 1995 the MD-effect was experimentally observed at the electron-proton collider HERA [13] on the level predicted in [10].

It was realized in last years that MD-effect in bremsstrahlung plays important role for the problem of beam lifetime. At storage rings TRISTAN and LEP-I, the process of a single bremsstrahlung was the dominant mechanism for the particle losses in beams. If electron loses more than 1 % of its energy, it leaves the beam. Since MD-effect reduced considerable the effective cross section of this process, the calculated beam lifetime in these storage rings was larger by about 25 % for TRISTAN [14] and by about 40 % for LEP-I [15] (in accordance with the experimental data) then without taken into account the MD-effect.

According to our calculations [16], at B-factories PEP-II and KEKB the MD-effect reduces beam losses due to bremsstrahlung by about 20%.

It is seen from this brief listing that the MD-effect is a phenomenon interesting from the theoretical point of view and important from the experimental point of view.

B. Essence of the Baier-Katkov paper

In recent paper [17], previous results [4], [5], [8] about bremsstrahlung spectrum had been revised. It was claimed that an additional "subtraction associated with the extraction of pure fluctuation process" has to be done. The reason to perform such an subtraction explained as follows: "At the beam collision the momentum transfer may arise due to interaction of the emitting particle with the opposite beam as a whole (due to coherent interaction with average field of the beam) and due to interaction with an individual particle of the opposite beam. Here we consider *the incoherent* process only (connected with the incoherent fluctuation of density) and so we have to subtract the coherent contribution". Analysis, performed in paper [17], results in the conclusion that this additional "subtraction term" in the spectrum is not important for the MD-1 experiment [1], but it should be taken into account in processing the HERA experiment [13]; it also may be important for the future experiments at linear e^+e^- colliders. It should be noted that in paper [17] there is no derivation of the starting formulae, and all physical reasons for such a subtraction were cited above. On the other hand, in paper [17] there is a general remark that their consideration was motivated by corresponding calculations for bremsstrahlung of ultra-relativistic electrons on oriented crystals.

In the present paper we analyze the coherent and incoherent contributions in the conditions, considered in paper [17], when the coherent length $l_{\rm coh}$ is much smaller than the bunch length l but much larger than the mean distance between particles a, i.e. $a \ll l_{\rm coh} \ll l$. We derive expressions for the coherent and incoherent contributions and show that under these conditions the coherent contribution is completely negligible and, therefore, there is no need to revise the previous results. This conclusion is quite natural. A usual bunch at colliders can be considered as a gaseous media with a smooth particle distribution which has characteristic scales of the order of bunch sizes. In particular, the average particle density in such a bunch has the only scale in the longitudinal direction — the length of the bunch l. Therefore, the average field of the bunch has the spectral components in the region of frequancies $\omega = q_z c \sim c/l_{\rm coh} \lesssim c/l$ and vanishes in the region of much higher frequencies considered here. On the contrary, in the crystal case there is another scale related to the size of the particle localization in the crystal structure. In this case, the additional subtraction should be taken into account for incoherent contribution. It seems that the electron radiation on oriented crystals played a misleading role for consideration of the MD-effect in [17]. To clarify a question we present our calculations in full details.

II. QUALITATIVE DESCRIPTION OF THE MD-EFFECT

Qualitatively we describe the MD–effect using as an example the $ep \to ep\gamma$ process[23]. This reaction is defined by the diagrams of Fig. 1 which describe the radiation of the photon by the electron (the contribution of the photon radiation by the proton can be neglected). The large impact parameters $\varrho \gtrsim \sigma_\perp$, where $\sigma_\perp$ is the transverse beam size, correspond to small momentum transfer $\hbar q_\perp \sim (\hbar/\varrho) \lesssim (\hbar/\sigma_\perp)$. In this region, the given reaction can be represented as a Compton scattering (Fig. 2) of the equivalent photon, radiated by the proton, on the electron. The equivalent photons with frequency ω form a "disk" of radius $\varrho_m \sim \gamma_p c/\omega$ where $\gamma_p = E_p/(m_p c^2)$ is the Lorentz-factor of the proton. Indeed, the electromagnetic field of the proton is γ_p–times contracted in the direction of motion. Therefore, at distance ϱ from the axis of motion a characteristic longitudinal length of a region occupied by the field can be estimated as $\lambda \sim \varrho/\gamma_p$ which leads to the frequency $\omega \sim c/\lambda \sim \gamma_p c/\varrho$.

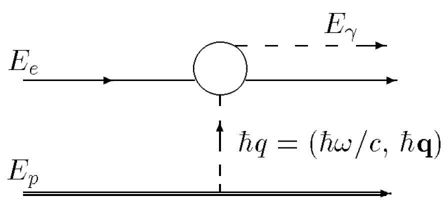

FIG. 1: Block diagram of radiation by the electron.

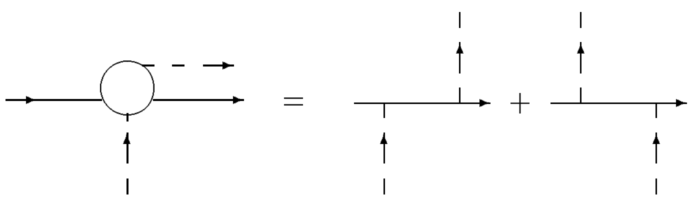

FIG. 2: Compton scattering of equivalent photon on the electron.

In the reference frame connected with the collider, the equivalent photon with energy $\hbar\omega$ and the electron with energy $E_e \gg \hbar\omega$ move toward each other (Fig. 3) and perform a Compton scattering. The characteristics of this process are well known. The main contribution to the Compton scattering is given by the region where the scattered photons fly in a direction opposite to that of the initial photons. For such a backward scattering, the energy of the equivalent photon $\hbar\omega$ and the energy of the final photon E_γ and its emission angle θ_γ are related by

$$\hbar\omega = \frac{E_\gamma}{4\gamma_e^2(1-E_\gamma/E_e)}\left[1+(\gamma_e\theta_\gamma)^2\right] \tag{1}$$

and, therefore,

$$\hbar\omega \sim \frac{E_\gamma}{4\gamma_e^2(1-E_\gamma/E_e)}\,. \tag{2}$$

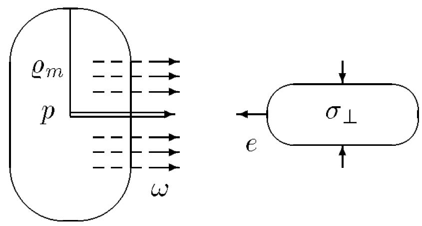

FIG. 3: Scattering of equivalent photons, forming the "disk" with radius ϱ_m, on the electron beam with radius $\sigma_\perp$.

As a result, we find the radius of the "disk" of equivalent photons with the frequency ω (corresponding to a final photon with energy E_γ) as follows:

$$\varrho_m = \frac{\gamma_p c}{\omega} \sim \lambda_e\, 4\gamma_e\gamma_p\, \frac{E_e - E_\gamma}{E_\gamma}\,, \quad \lambda_e = \frac{\hbar}{m_e c} = 3.86\cdot 10^{-11}\ \text{cm}\,. \tag{3}$$

For the HERA collider with $E_p = 820$ GeV and $E_e = 28$ GeV one obtains

$$\varrho_m \gtrsim 1\ \text{cm} \quad \text{for} \quad E_\gamma \lesssim 0.2\ \text{GeV}\ . \tag{4}$$

Equation (3) is also valid for the $e^-e^+ \to e^-e^+\gamma$ process with replacement protons by positrons. For the VEPP-4 collider it leads to

$$\varrho_m \gtrsim 1\ \text{cm} \quad \text{for} \quad E_\gamma \lesssim 15\ \ \text{MeV}\ , \tag{5}$$

for the PEP-II and KEKB colliders we have

$$\varrho_m \gtrsim 1\ \text{cm} \quad \text{for} \quad E_\gamma \lesssim 0.1\ \ \text{GeV}\ . \tag{6}$$

The standard calculation corresponds to the interaction of the photons forming the "disk" with the unbounded flux of electrons. However, the particle beams at the HERA collider have finite transverse beam sizes of the order of $\sigma_\perp \sim 10^{-2}$ cm. Therefore, the equivalent photons from the region $\sigma_\perp \lesssim \varrho \lesssim \varrho_m$ cannot interact with the electrons from the other beam. This leads to the decreasing number of the observed photons and the "observed cross section" $d\sigma_{\rm obs}$ is smaller than the standard cross section $d\sigma$ calculated for an infinite transverse extension of the electron beam,

$$d\sigma_{\rm obs} = d\sigma - d\sigma_{\rm cor}. \tag{7}$$

Here the correction $d\sigma_{\rm cor}$ can be presented in the form

$$d\sigma_{\rm cor} = d\sigma_{\rm C}(\omega,\, E_\gamma)\; dn(\omega) \tag{8}$$

where $dn(\omega)$ denotes the number of "missing" equivalent photons and $d\sigma_{\rm C}$ is the cross section of the Compton scattering. Let us stress that the equivalent photon approximation in this region has a high accuracy (the neglected terms are of the order of $1/\gamma_p$). But for the qualitative description it is sufficient to use the logarithmic approximation in which this number is (see[18], §99)

$$dn = \frac{\alpha}{\pi}\frac{d\omega}{\omega}\frac{dq_\perp^2}{q_\perp^2}\,. \tag{9}$$

Since $q_\perp \sim 1/\varrho$, we can present the number of "missing" equivalent photons in the form

$$dn = \frac{\alpha}{\pi}\frac{d\omega}{\omega}\frac{d\varrho^2}{\varrho^2} \tag{10}$$

with the integration region in ϱ:

$$\sigma_\perp \lesssim \varrho \lesssim \varrho_m = \frac{\gamma_p c}{\omega}\,. \tag{11}$$

As a result, this number is equal to

$$dn(\omega) = 2\frac{\alpha}{\pi}\frac{d\omega}{\omega}\ln\frac{\varrho_m}{\sigma_\perp}\,, \tag{12}$$

and the correction to the standard cross section with logarithmic accuracy is[24]

$$d\sigma_{\rm cor} = \frac{16}{3}\alpha r_e^2\,\frac{dy}{y}\,\left(1-y+\tfrac{3}{4}y^2\right)\ln\frac{4\gamma_e\gamma_p(1-y)\lambda_e}{y\sigma_\perp}\,,\;\; y=\frac{E_\gamma}{E_e}\,. \tag{13}$$

III. APPROXIMATIONS

For future linear e^+e^- colliders the transverse sizes of the beams will change significantly during the time of interaction due to a mutual attraction of very dense beams. However, for most of the ordinary accelerators, including practically all e^+e^- and ep storage rings, the change of the transverse beam sizes during the collisions can be neglected. Below we use two main approximations: 1) the particle movement in the bunches has a quasi-classical character; 2) the particle distribution remains practically unchanged during the collision. Besides, in calculating the coherent contribution, we neglect correlations in particle coordinates. All these approximation just the same as in Ref. [17]. For definiteness, we use again the ep collisions as an example.

Therefore, if the proton (electron) bunch moves along (opposite) the direction of z-axis with the velocity v_p (v_e), its density has the form

$$n_p = n_p(\boldsymbol{\varrho}, z - v_p t)\,, \quad n_e = n_e(\boldsymbol{\varrho}, z + v_e t)\,. \tag{14}$$

We also introduce so called "transverse densities"

$$n_p(\boldsymbol{\varrho}) = \int n_p\, dz\,, \quad n_e(\boldsymbol{\varrho}) = \int n_e\, dz \tag{15}$$

which is equal to the total number of protons (electrons) which cross a unit area around the impact parameter $\boldsymbol{\varrho}$ during the collision.

Below we consider in detail the case when an electron deflection angle θ_e is smaller than the typical radiation angle $\sim 1/\gamma_e$. It is easy to estimate the ratio of these angles. The electric $\mathbf{E}$ and magnetic $\mathbf{B}$ fields of the proton bunch are approximately equal in magnitude, $|\mathbf{E}| \approx |\mathbf{B}| \sim eN_p/(\sigma_x + \sigma_y)$. These fields are transverse and they deflect the electron into the same direction. In such fields the electron moves around a circumference of radius $R \sim \gamma_e m_e c^2/(eB)$ and gets the deflection angle $\theta_e \sim l/R$. Therefore, the ratio of these angles is of the order of

$$\frac{\theta_e}{(1/\gamma_e)} \sim \eta = \frac{r_e N_p}{\sigma_x + \sigma_y}\,. \tag{16}$$

The parameter $\eta \gg 1$ only for the SLC and future linear e^+e^- colliders, in most of the colliders $\eta \lesssim 1$.

In our consideration we use the equivalent photon approximation. In the region of interest (where impact parameters are large, $\varrho \gtrsim \sigma_\perp$) this simple and transparent method has a high accuracy. On the other hand, the operator quasi-classical method, used in Ref. [17], just coincides in this region with the equivalent photon approximation.

IV. COHERENT AND INCOHERENT CONTRIBUTIONS

A. General formulae

The corresponding formulae for the number of events in a single collision of the electron and proton bunches can be found in papers [19], [20]. To calculate the MD-effect, we need to know the distribution of equivalent photons (EP) for large values of impact parameters. In this region we can consider the electron–proton scattering as the scattering of electrons on the electromagnetic field of the proton bunch. Replacing this field by the flux of EP with some energy distribution, we obtain the number of the produced photons in the form

$$dN_\gamma = dL_{\gamma e}(\omega)\ d\sigma_{\rm C}(\omega,\ E_\gamma); \quad dL_{\gamma e}(\omega) = n_\gamma(\boldsymbol{\varrho},\omega) d\omega\, n_e(\boldsymbol{\varrho})\, d^2\varrho\,. \tag{17}$$

Here $n_e(\boldsymbol{\varrho})$ is the transverse electron density and $n_\gamma(\boldsymbol{\varrho},\omega)d\omega$ is the transverse density of EP with the frequencies in the interval from ω to $\omega + d\omega$. The quantity $dL_{\gamma e}(\omega)$ denotes the differential luminosity for the collisions of EP and electrons and $d\sigma_{\rm C}(\omega, E_\gamma)$ is the Compton cross section for the scattering of the equivalent photon with the frequency ω on the electron.

The transverse density of the EP is

$$n_\gamma(\boldsymbol{\varrho},\omega)\ d\omega = \frac{c}{4\pi^2}\ \left\langle \mid \mathbf{E}_\omega(\boldsymbol{\varrho})\ \mid^2 \right\rangle\ \frac{d\omega}{\hbar\omega} \tag{18}$$

where $\mathbf{E}_\omega(\boldsymbol{\varrho})$ is the spectral component of the collective electric field of the proton bunch. The sign $\langle \dots \rangle$ denotes the averaging over fluctuations of the field connected, for example, with the fluctuations of particle positions for many collisions of bunches in a given experiment. This field depends on a distribution of charges in the proton bunch at $t = 0$. We introduce the exact (fluctuating) density of the proton bunch $n(\mathbf{r})$ and the averaging density

$$n_p(\mathbf{r}) = \langle n(\mathbf{r})\rangle \tag{19}$$

as well as their form factors

$$F(\mathbf{q}) = \int n(\mathbf{r})\, {\rm e}^{-{\rm i}\mathbf{qr}}\ d^3r\,,\quad F_p(\mathbf{q}) = \langle F(\mathbf{q})\rangle = \int n_p(\mathbf{r})\, {\rm e}^{-{\rm i}\mathbf{qr}}\ d^3r \tag{20}$$

with the normalization

$$F(0) = \int n(\mathbf{r})\, d^3r = F_p(0) = N_p\ . \tag{21}$$

In the classical limit

$$n(\mathbf{r}) = \sum_a \delta(\mathbf{r} - \mathbf{r}_a)\,, \quad F(\mathbf{q}) = \sum_a \mathrm{e}^{-i\mathbf{q}\mathbf{r}_a} \tag{22}$$

where $\mathbf{r}_a$ is the radius-vector of the a-th proton. In these notations, the exact (fluctuating) collective field is

$$\mathbf{E}_\omega(\boldsymbol{\varrho}) = -\frac{\mathrm{i}e}{\pi c} \int \frac{\mathbf{q}_\perp\, \mathrm{e}^{\mathrm{i}\mathbf{q}_\perp \boldsymbol{\varrho}}}{\mathbf{q}_\perp^2 + \omega^2/(c\gamma_p)^2}\, F(\mathbf{q})\, d^2q_\perp \tag{23}$$

with $q_z = \omega/c$.

As a result, the number of events

$$dN_\gamma \propto n_\gamma(\boldsymbol{\varrho}, \omega) = \frac{\alpha}{4\pi^4\omega} \int \frac{(\mathbf{q}_\perp \mathbf{q}'_\perp)\, \mathrm{e}^{\mathrm{i}(\mathbf{q}_\perp - \mathbf{q}'_\perp)\boldsymbol{\varrho}}}{[\mathbf{q}_\perp^2 + \omega^2/(c\gamma_p)^2][(\mathbf{q}'_\perp)^2 + \omega^2/(c\gamma_p)^2]}\, A\, d^2q_\perp\, d^2q'_\perp \tag{24}$$

depends on the quantity

$$A = \langle\, F(\mathbf{q})\, F^*(\mathbf{q}')\, \rangle = \int \langle\, n(\mathbf{r})\, n(\mathbf{r}')\, \rangle\ \mathrm{e}^{-\mathrm{i}(\mathbf{q}\mathbf{r} - \mathbf{q}'\mathbf{r}')}\, d^3r\, d^3r' \tag{25}$$

in which

$$q_z = q'_z = \omega/c\,. \tag{26}$$

B. Coherent and incoherent bremsstrahlung

The obtained general formulae include the coherent and incoherent contributions. The coherent contribution is determined by the average field which is given by Eq. (23) with the replacement $F(\mathbf{q})$ by $F_p(\mathbf{q})$ or with the replacement of the exact density by the average density. The averaged density of the proton bunch has a single scale in the longitudinal direction — the length of the bunch l. Therefore, the average field of the bunch is essential in the region of frequencies $\omega = cq_z \lesssim c/l$ and should be small in the region of large frequencies $\omega \gg c/l$. In particular, if the proton bunch has the Gaussian distribution, its form factor is equal to

$$F_p(\mathbf{q}) = N_p \exp\left[-\tfrac{1}{2}(q_x\sigma_x)^2 - \tfrac{1}{2}(q_y\sigma_y)^2 - \tfrac{1}{2}(\omega l/c)^2\right] \tag{27}$$

and vanishes in the discussed region of frequencies from the interval $c/l \ll \omega \ll c/a$ where a is the mean distance between particles.

A bunch at colliders can be treated as a continues media with a smooth average particle distribution of the Caussian type. If we neglect the correlations of the particle coordinates in such media, the average product of densities $\langle n(\mathbf{r})\, n(\mathbf{r}')\rangle$ is expressed only via the average densities as follows (see, for example, [21])

$$\langle n(\mathbf{r})\, n(\mathbf{r}')\rangle = n_p(\mathbf{r})\, n_p(\mathbf{r}') + \delta(\mathbf{r}-\mathbf{r}')\, n_p(\mathbf{r})\,. \tag{28}$$

As a result, the quantity A can be presented as a sum of coherent and incoherent contributions:

$$A = A_{\text{coh}} + A_{\text{incoh}}\,. \tag{29}$$

The coherent contribution is related to the first item $n_p(\mathbf{r})\, n_p(\mathbf{r}')$ in Eq. (28) and is equal to

$$A_{\text{coh}} = F_p(\mathbf{q})\, F_p^*(\mathbf{q}')\,. \tag{30}$$

This formula was used in Refs. [19], [20] to obtain main characteristics of the coherent bremsstrahlung. It also allows us to obtain the following estimate for the Gaussian beams in the region of interest (at $|q_x| \lesssim 1/\sigma_x$ and $|q_y| \lesssim 1/\sigma_y$):

$$A_{\text{coh}} \sim N_p^2 \exp\left[-(\omega l/c)^2\right]\,. \tag{31}$$

The incoherent contribution is connected with the second item $\delta(\mathbf{r}-\mathbf{r}')\, n_p(\mathbf{r})$ in Eq. (28) and is equal to (taking into account Eq. (26))

$$A_{\text{incoh}} = F(\mathbf{q}_\perp - \mathbf{q}'_\perp)\,. \tag{32}$$

Note, that this expression is determined by the transverse average density of the proton bunch and it does not depend on ω. For the Gaussian beams in the region of interest, we obtain from (32) an estimate

$$A_{\text{incoh}} \sim N_p\,. \tag{33}$$

Formula (32) was used to derive the previous results about MD-effect (for details see review [2]). It is seen from the above consideration that the incoherent contribution for usual colliding beams has no “additional subtraction” related to the average electromagnetic field of the proton bunch.

C. Comparison with the approach used in [17]

We derive the final expression for the incoherent contribution from general equations (17), (18) and (23) as a simple consequence of natural assumptions about the particle distribution in a proton bunch. It is useful to rewrite these equations in the form convenient for comparison with the corresponding equations in [17]. To do this, we note that the Compton cross section $d\sigma_{\rm C}(\omega, E_\gamma) \propto |\mathbf{e}\mathbf{M}_{\rm C}|^2$ where $\mathbf{e}\mathbf{M}_{\rm C}$ is the amplitude of the Compton scattering for the EP with the polarization vector $\mathbf{e}$. Therefore, the number of events

$$dN_\gamma \propto |M|^2\,, \quad M = \mathbf{E}_\omega(\boldsymbol{\varrho})\mathbf{M}_{\rm C} \tag{34}$$

where the quantity M is proportional to the probability amplitude of the process. Further, we use Eq. (22) and the well known equality

$$\int \frac{\mathbf{q}_\perp\, \mathrm{e}^{\mathrm{i}\mathbf{q}_\perp \boldsymbol{\varrho}}}{\mathbf{q}_\perp^2 + (1/b)^2}\, d^2 q_\perp = \frac{2\pi \mathrm{i}}{b}\, \frac{\boldsymbol{\varrho}}{\varrho}\, K_1(\varrho/b) \tag{35}$$

where $K_n(x)$ denotes the modified Bessel function of third kind with integer index n (McDonald function). Then we obtain

$$M = \sum_{a=1}^{N_p} M_a\,, \quad M_a = \mathbf{E}_\omega^{(a)}(\boldsymbol{\varrho})\,\mathbf{M}_{\rm C}\,, \quad \mathbf{E}_\omega^{(a)}(\boldsymbol{\varrho}) = -\frac{2e}{c\varrho_m}\,\frac{\boldsymbol{\varrho}'_a}{\varrho'_a} K_1\left(\varrho'_a/\varrho_m\right)\,\mathrm{e}^{-\mathrm{i}\omega z_a/c} \tag{36}$$

where $\boldsymbol{\varrho}'_a = \boldsymbol{\varrho}_a - \boldsymbol{\varrho}$ is the impact parameter between the a-th proton and the electron and the parameter $\varrho_m = \gamma_p c/\omega$ is the radius of the "disc" of EP (see Fig. 3). The item M_a is the contribution to M related to the interaction of the electron with the a-th proton.

Our expression for M_a coincides with the corresponding expression in [17] with the only (but essential!) exception: in paper [17] the factor

$$\mathrm{e}^{-\mathrm{i}\omega z_a/c} \tag{37}$$

is absent. It means that in Ref. [17] it was neglected by the dependence of the exact and average density of the proton bunch on longitudinal coordinates. However, according to the consideration given above, this dependence is crucial for description of the coherent contribution.

Let us clarify this point by the following simple calculations. Our incoherent contribution corresponds to the average square of the individual fields of protons, i.e. to the sum

$$S = \sum_a \langle\, |M_a|^2\, \rangle \tag{38}$$

in which the expression $|M_a|^2$ does not depend on z_a. In that case the sum over a transforms to the following integral over the transverse coordinates

$$\sum_a \langle \ldots \rangle \to \int d^2\varrho_a dz_a\, n_p(\boldsymbol{\varrho}_a, z_a)\, \ldots = \int d^2\varrho_a\, n_p(\boldsymbol{\varrho}_a)\, \ldots\, . \tag{39}$$

After the substitution $\boldsymbol{\varrho}_a = \boldsymbol{\varrho} + \mathbf{r}_\perp$, we obtain the expression

$$S = \frac{4e^2}{c^2\varrho_m^2}\, |\mathbf{e}\mathbf{M}_\mathrm{C}|^2 \int n_p(\boldsymbol{\varrho} + \mathbf{r}_\perp)\, K_1^2(r_\perp/\varrho_m)\, d^2 r_\perp \tag{40}$$

which is equivalent to the previous result (32).

It was claimed in paper [17] that an additional subtraction S_1 has to be done which corresponds to the square of average individual fields of protons, i.e.

$$S \to S - S_1\, , \quad S_1 = \sum_a |\langle\, M_a\, \rangle|^2\, . \tag{41}$$

As we can jude from the final expression for S_1, the averaging in this case means averaging over the transverse coordinates of protons only,

$$\langle\, M_a\, \rangle = \int M_a\, \frac{n_p(\boldsymbol{\varrho}_a)}{N_p}\, d^2\varrho_a\, . \tag{42}$$

It would be natural if the expression M_a would not depend on the longitudinal coordinates of protons z_a.

But according to our analysis, it is not the case since

$$M_a \propto \mathrm{e}^{-\mathrm{i}\omega z_a/c}\, . \tag{43}$$

Taking into account this very fact, one has to perform averaging over the longitudinal coordinates z_a as well, i.e. instead of (42) we have

$$\langle\, M_a\, \rangle = \int M_a\, \frac{n_p(\boldsymbol{\varrho}_a, z_a)}{N_p}\, d^2\varrho_a dz_a\, . \tag{44}$$

It changes the final result dramatically, since in the discussed region of frequencies $\omega \gg c/l$ the expression $\langle\, M_a\, \rangle$ disappears. For example, in the case of the Caussian distribution we have

$$\langle\, M_a\, \rangle \propto \exp\left[-\tfrac{1}{2}(\omega l/c)^2\right] \ll 1 \quad \text{at} \quad \omega \gg c/l\, . \tag{45}$$

V. CONCLUSIONS

Let us compare the coherent and incoherent contributions for the Gaussian beams. In this case, the ratio

$$\frac{A_{\rm coh}}{A_{\rm incoh}} \sim N_p \exp\left[-(\omega l/c)^2\right] \tag{46}$$

is determined by the parameters $\omega l/c$. Since $\hbar\omega \sim E_\gamma/[4\gamma_e^2(1-E_\gamma/E_e)]$, it is also useful to introduced the coherence length

$$l_{\rm coh} = \frac{4\gamma_e^2 \hbar c}{E_\gamma}\,(1-E_\gamma/E_e) \tag{47}$$

and the critical energy for the coherent bremsstrahlung

$$E_c = \frac{4\gamma_e^2 \hbar c}{l}\,. \tag{48}$$

If the coherence length is large, $l_{\rm coh} \gtrsim l$, or if the final photon energy is small, $E_\gamma \lesssim E_c$, the parameter $\omega l/c \lesssim 1$ and the coherent contribution is dominant.

On the contrary, in the region of large photon energy, $E_\gamma \gg E_c$, or small coherence length, $l_{\rm coh} \ll l$, considered in paper [17], the incoherent contribution dominates. In particular, at $\omega l/c > 6$ and $N_p \sim 10^{11}$, the ratio

$$\frac{A_{\rm coh}}{A_{\rm incoh}} \sim N_p\, \mathrm{e}^{-36} \ll 1\,, \tag{49}$$

the coherent contribution is completely negligible and the previous formulae for the MD-effect are valid.

This consideration shows that the effect, derived in paper [17], is absent just in the region, discussed in this paper. At the end of this section we reconsider the experiments analyzed in paper [17].

The HERA experiment [13]. In this case $E_e = 27.5$ GeV and $l = 8.5$ cm, therefore, $E_c = 27$ keV. For the observed photon energies $E_\gamma = 2 \div 8$ GeV, the parameter

$$\frac{\omega l}{c} \sim \frac{E_\gamma}{E_c} > 10^4\,, \tag{50}$$

and the coherent contribution is completely negligible[25]. Therefore, the new correction to the previous results on the level of 10 %, obtained in [17], is, in fact, absent.

The VEPP-4 experiment [1]. In this case $E_e = 1.84$ GeV and $l = 3$ cm, therefore, $E_c = 0.34$ keV. For the observed photon energies $E_\gamma \gtrsim 1$ MeV, the parameter

$$\frac{\omega l}{c} \sim \frac{E_\gamma}{E_c} > 10^3\,, \tag{51}$$

and the coherent contribution is completely negligible.

The case of a "typical linear collider" with $E_e = 500$ GeV and $E_\gamma = E_e/1000$. This example, considered in paper [17], is irrelevant for the discussed problem, since the coherent radiation (or beamstrahlung) at a typical linear collider is absolutely dominated in this very region over the ordinary incoherent bremsstrahlung — see, for example, the TESLA project [22].

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[23] Below we use the following notations: N_e and N_p are the numbers of electrons and protons (positrons) in the bunches, $\sigma_z = l$ is the longitudinal, σ_x and σ_y are the horizontal and vertical transverse sizes of the proton (positron) bunch, $\gamma_e = E_e/(m_e c^2)$, $\gamma_p = E_p/(m_p c^2)$ and $r_e = e^2/(m_e c^2)$ is the classical electron radius.

[24] Within this approximation, the standard cross section has the form

$$d\sigma = d\sigma_{\mathrm{C}} \frac{\alpha}{\pi} \frac{d\omega}{\omega} \frac{dq_\perp^2}{q_\perp^2} = \frac{16}{3} \alpha r_e^2 \, \frac{dy}{y} \, \left(1 - y + \tfrac{3}{4} y^2\right) \ln \frac{4\gamma_e \gamma_p (1-y)}{y}$$

with the integration region $\hbar\omega/(c\gamma_p) \lesssim \hbar q_\perp \lesssim m_e c$ corresponding to the impact parameters ϱ in the interval $\lambda_e \lesssim \varrho \lesssim \varrho_m$.

[25] Moreover, in the HERA experiment the coherence length is of the order of the mean distance between particles in the proton bunch, $l_{\mathrm{coh}} \sim a \sim (l \sigma_x \sigma_y / N_p)^{1/3}$.